Neutrino Masses from Non-minimal Gravitational Interactions of Massive Neutral Fermions

Abhinav Gupta[*], Namit Mahajan[†] and Amitabha Mukherjee[‡]

Department of Physics and Astrophysics,

University of Delhi, Delhi-110 007, India.

Abstract

A new mechanism is proposed for generating neutrino masses radiatively through a non-minimal coupling to gravity of fermionic bilinears involving massive neutral fermions. Such coupling terms can arise in theories where the gravity sector is augmented by a scalar field. They necessarily violate the principle of equivalence, but such violations are not ruled out by present experiments. It is shown that the proposed mechanism is realised most convincingly in theories of the Randall- Sundrum type, where gravity couples strongly in the TeV range. The mechanism has the potential for solving both the solar and atmospheric neutrino problems. The smallness of neutrino masses in this scenario is due to the fact that the interaction of the massive neutral fermions arises entirely from higher-dimensional operators in the effective Lagrangian.

Keywords: Neutrino Masses, Non-minimal coupling, massive neutral fermion

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Introduction

It is well known that non-zero neutrino masses can result in neutrino oscillations. There seems to be strong experimental evidence for neutrino oscillations [1]. Several models have been proposed to generate the neutrino mass matrix [2]. Most of these models modify the particle content of the Standard Model (SM) and introduce new fields and/or new interactions. Possibility of generating neutrino masses in theories with extra dimensions have also been considered [3]. In all these scenarios gravity couples universally to matter, preserving the Equivalence Principle (EP).

The EP has been tested on macroscopic scales. This does not rule out possible deviations from this principle at very short distances. The possibility of violation of the Equivalence Principle (VEP) as a source of neutrino oscillations has been explored in the literature [4]. These studies assume non-universal coupling of gravity to different

*E–mail : abh@ducos.ernet.in

†E–mail : nm@ducos.ernet.in, nmahajan@physics.du.ac.in

‡E–mail : am@ducos.ernet.in

In this paper, we explore the possibility of generating neutrino masses through a violation (weak and at very short distances) of EP by introducing non-minimal gravitational interaction terms in the action involving neutrinos and Massive Neutral Fermions (MNFs). We show that it is possible to generate neutrino masses by the addition of such a non-minimal term in theories where the gravity sector is enlarged by additional massive scalar field(s). The TeV scale quantum gravity theories [5] are found to be the natural candidates for realising this mechanism.

The Model

Consider a theory of gravity having a scalar field coupling to the matter fields through the trace of the energy-momentum tensor. The total action involving gravitation and matter coupling minimally is

$$S_{total} = S_{gravity} + \int d^4x \, \sqrt{-g} \, \mathcal{L}_{matter} \tag{1}$$

The matter Lagrangian is constructed so that in addition to the usual SM fields there is a MNF (S_1) with Dirac mass (m_{S_1}) and the kinetic terms for the right handed partners (ν_R^i) of the SM neutrinos with the index i running over neutrino flavours e, μ, τ. The MNF and the right handed neutrinos are $SU(2)_L \times U(1)_Y$ singlets thus having no SM interactions. The only non-SM interactions are in the gravity sector, induced by a non-minimal coupling leading to an additional term in the action of the form

$$S_{NM} = -\frac{1}{\Lambda^2} \sum_i f^i \int d^4x \sqrt{-g} R \left[\, \bar{\psi^i}_L \tilde{\Phi} S_{1R} - c^i \frac{v}{\sqrt{2}} \bar{S}_{1L} \nu_R^i + h.c \, \right] \tag{2}$$

where ψ_L^i is the i^{th} SM lepton doublet, $\tilde{\Phi} = \iota \Phi^*$ is the conjugate Higgs doublet, v is the Higgs vacuum expectation value (VEV) and R is the Ricci scalar. The coefficients f^i and c^i are arbitrary dimensionless parameters determining the strengths with which different flavours couple in the above non-minimal fashion. Λ is the characteristic mass-scale of the theory of gravity under consideration, determined by some higher theory which becomes relevant above this scale. This term is invariant under $SU(2)_L \times U(1)_Y$ transformations and violates EP. The extent of VEP by different neutrino flavours is characterised by f^i and c^i. Such higher order non-minimal terms may be generated as a result of quantum effects of the higher theory of which this theory is a low-energy manifestation. This additional term leads to a modification of the energy-momentum tensor

$$\Delta \, T_{\mu\nu} = \frac{-2}{\Lambda^2} \sum_i f^i (\eta_{\mu\nu} \partial_\alpha \partial^\alpha - \partial_\mu \partial_\nu) \left[\, \bar{\psi^i}_L \tilde{\Phi} S_R - c^i \frac{v}{\sqrt{2}} \bar{S}_L \nu_R^i + h.c. \, \right] \tag{3}$$

Assuming the coupling of the scalar field (ϕ with mass M_ϕ) to the trace of energy-momentum tensor is of the form

$$\mathcal{L}_{int} = -\frac{\phi}{\Lambda} (T_{matter})^\mu_\mu \tag{4}$$

action of the scalar to neutrinos of the form

$$\mathcal{L}_{int} = \sum_i \frac{6 f^i M_\phi^2 v}{\sqrt{2}\Lambda^3} \left[\bar{\nu^i}_L S_{1R} - c^i \bar{S}_{1L} \nu^i_R + h.c \right] \phi \tag{5}$$

For reasons evident later, we choose the parameters c^i to be all equal to some constant c. From the interaction Lagrangian it is evident that a neutrino mass matrix would be induced by the radiative process shown in Figure 1.

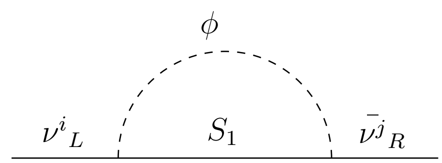

Figure 1: Typical Feynman diagram (evaluated at zero external momenta) contributing to the ij entry of the mass matrix.

The mass matrix for three generations has the following structure

$$\mathcal{M}_\nu = \delta \begin{pmatrix} f^{e2} & f^e f^\mu & f^e f^\tau \\ f^e f^\mu & f^{\mu 2} & f^\mu f^\tau \\ f^e f^\tau & f^\mu f^\tau & f^{\tau 2} \end{pmatrix} \tag{6}$$

where

$$\delta = \frac{c\ m_S}{16\pi^2} \left(\frac{6\ M_\phi^2\ v}{\sqrt{2}\ \Lambda^3} \right)^2 \left(\frac{1}{M_\phi^2 - m_S^2} \right) \left[M_\phi^2 \log\left(\frac{\Lambda^2}{M_\phi^2} \right) - m_S^2 \log\left(\frac{\Lambda^2}{m_S^2} \right) \right]$$

This matrix has two zero eigenvalues and one non-zero eigenvalue m_3, given by

$$m_3 = \delta(f^{e2} + f^{\mu 2} + f^{\tau 2}) \tag{7}$$

The weak interaction gauge eigenstates ν^i are related to the mass eigenstates ν^a ($a = 1,2,3$) by the mixing matrix U as

$$\nu^i = \sum_a U^{ia} \nu^a$$

where the mixing matrix is

$$U = \begin{pmatrix} s_\alpha c_\theta + c_\alpha s_\theta \gamma & -s_\alpha s_\theta + c_\alpha c_\theta \gamma & \rho c_\alpha \gamma \\ -c_\alpha c_\theta + s_\alpha s_\theta \gamma & c_\alpha s_\theta + s_\alpha c_\theta \gamma & \rho s_\alpha \gamma \\ -\rho s_\theta \gamma & -\rho c_\theta \gamma & \gamma \end{pmatrix} \tag{8}$$

where c and s with subscript are the cosine and sine of the subscript respectively, $\rho = (f^{e2} + f^{\mu 2})^{\frac{1}{2}}/f^\tau$, $\tan\alpha = f^\mu/f^e$ and $\gamma = 1/(1+\rho^2)^{\frac{1}{2}}$. The angle θ corresponds to possible rotations in the degenerate ν^1-ν^2 sub-space.

The above analysis assumes the same value c for the parameters c^i as mentioned trum as such since one still gets two zero eigenvalues, the non-zero eigenvalue, though, is altered. In this minimal framework, the atmospheric neutrino problem is solved by $\nu^\mu \to \nu^\tau$ oscillations, with the assumption that $f^e << f^\mu, f^\tau$ implying that $\nu^e - \nu^\mu$ and $\nu^e - \nu^\tau$ mixing is negligible compared to $\nu^\mu - \nu^\tau$ mixing.

To solve both the atmospheric and solar neutrino problems in a three flavour framework, we need two mass scales corresponding to the different length scales involved in the two problems. Therefore, the degeneracy of the ν^1-ν^2 sub-space has to be lifted. This can be achieved by extending this minimal framework by including two more MNFs (S_2 and S_3) having a mass similar to m_{S_1}. We thus have three MNFs S_m ($m = 1, 2, 3$) interacting non-minimally to generate the neutrino mass matrix. The Lagrangian for the extended system is

$$\mathcal{L}_{int} = \frac{6M_\phi^2 v}{\sqrt{2}\Lambda^3} \sum_{m=1}^{3} \sum_{i=e,\mu,\tau} \left[f_m^i (\bar{\nu^i}_L S_{mR} - c_m^i \bar{S}_{mL} \nu_R^i) + h.c. \right] \phi \tag{9}$$

For simplicity we choose $c_m^i = 1$ and the masses for S_m to be the same.

The following choice of parameters

$$\begin{aligned} f_1^\tau = -f_1^\mu &= \frac{1}{\sqrt{2}} f_1^e \\ f_2^\tau = -f_2^\mu &= -\frac{1}{\sqrt{2}} f_2^e \\ f_3^\tau = f_3^\mu \quad &and \quad f_3^e = 0 \end{aligned} \tag{10}$$

reduces the mass matrix to the form

$$\mathcal{M}_\nu = \delta \begin{pmatrix} f_1^{e2} + f_2^{e2} & \frac{1}{\sqrt{2}}(f_2^{e2} - f_1^{e2}) & \frac{1}{\sqrt{2}}(f_1^{e2} - f_2^{e2}) \\ \frac{1}{\sqrt{2}}(f_2^{e2} - f_1^{e2}) & \frac{1}{2}(f_1^{e2} + f_2^{e2}) + f_3^{\mu 2} & -\frac{1}{2}(f_1^{e2} + f_2^{e2}) + f_3^{\mu 2} \\ \frac{1}{\sqrt{2}}(f_1^{e2} - f_2^{e2}) & -\frac{1}{2}(f_1^{e2} + f_2^{e2}) + f_3^{\mu 2} & \frac{1}{2}(f_1^{e2} + f_2^{e2}) + f_3^{\mu 2} \end{pmatrix} \tag{11}$$

with eigenvalues

$$m_1 = 2\delta f_1^{e2} \quad m_2 = 2\delta f_2^{e2} \quad m_3 = 2\delta f_3^{\mu 2} \tag{12}$$

The mixing matrix for this choice of parameters is

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{13}$$

This mixing matrix corresponds to the *'bimaximal mixing'* solution [6] to the solar and atmospheric neutrino problems in which the ν^e- ν^τ mixing angle is taken to be zero and the ν^e-ν^μ and ν^μ-ν^τ angles are taken to be equal to $\pi/4$. In general, the parameters can be altered to accommodate other possible solutions in a three flavour framework.

The *bimaximal mixing* solution is consistent with the following constraints [6] on the mass eigenvalues with the hierarchy $m_1 < m_2 < m_3$

$$\begin{aligned} \Delta_{atm.} &= m_3^2 - m_2^2 \sim 3.5 \times 10^{-3}\ eV^2 \\ \Delta_{sol.} &= m_2^2 - m_1^2 \sim 10^{-5}\ eV^2\ (Large Angle MSW solution) \end{aligned} \tag{14}$$

the masses of MNFs and the parameters f_m which are a measure of the extent of VEP. A large value of m_S would result in weaker violation of EP to be consistent with equation (14).

The formalism assumes the existence of a massive scalar in the gravity sector coupling to the trace of the matter energy-momentum tensor. This requirement is fulfilled in the current TeV scale gravity theories [5]. In the Randall-Sundrum (RS) scenario, a massive scalar, the radion, couples to the matter fields in the desired fashion making it a possible candidate to realise this kind of mass generation for neutrinos. In this scenario, $\Lambda \sim 10$ TeV and $M_\phi \sim 250$ GeV are typical scales, leading to $\delta \sim m_S \times 10^{-10}$. Equation (14) then leads to

$$\begin{aligned} f_2^{e4} - f_1^{e4} &\sim 10^{-8} \\ f_3^{\mu 4} - f_2^{e4} &\sim 10^{-6} \qquad for \;\; m_S \sim 100 \; GeV \end{aligned} \tag{15}$$

and

$$\begin{aligned} f_2^{e4} - f_1^{e4} &\sim 10^{-10} \\ f_3^{\mu 4} - f_2^{e4} &\sim 10^{-8} \qquad for \;\; m_S \sim 1 \; TeV \end{aligned} \tag{16}$$

It is evident that a larger mass for MNFs leads to a weaker VEP.

Discussion

The above analysis has been carried out in a three neutrino flavour framework with bi-maximal mixing parameters. If LSND results are confirmed in future, it would imply possible existence of a light sterile neutrino. Such a sterile neutrino would interact with the MNFs in a manner similar to ordinary neutrinos, acquiring mass and generating a third mass scale accounting for the LSND results (if confirmed).

In this formalism, since the mass matrix is generated only through non-standard gravitational interactions of the massive neutral fermions and ordinary neutrinos, the SM precision tests are not disturbed below the characteristic scale, Λ, of the effective theory. The constraints on the parameters have been obtained for the RS scenario, but the formalism applies to a generic class of scalar-tensor theories of gravitation in which the scalar couples to the trace of the energy-momentum tensor of matter.

It is evident that such a mass generation is not possible if the scalar field is massless. The non-minimal term would also give rise to interactions of neutrinos and MNFs with ordinary massless gravitons. But to lowest order in the effective coupling, the masslessness of the graviton forbids such an interaction. The non-zero mass of the scalar field ensures that the non-minimal interaction does not lead to any long-distance corrections to ordinary gravitational interactions.

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